rotating in the opposite direction from its neighbors. The qualitative flow patterns obtained were independent of the turbulence model used, although as one might expect, quantitative differences did exist between the S-A model and the RSM results. Although the present study was limited to a NACA 0015 plan form, it appears likely that, in general, these multiple vortex structures can be expected to form over most flat end-cap wings oriented at moderate angles of attack.

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Predictor–Corrector Approach for the Analysis of Sandwich Panels

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I. Introduction

THE through-the-thickness distribution of displacements and stresses of sandwich panels can be evaluated with multiple three-dimensional finite elements in the thickness direction[1,2] or with two-dimensional finite element models that allow higher-order shear deformations.[3] However, these approaches require substantial computational cost. More cost-effective approaches for the evaluation of stresses include postprocessing-type procedures that utilize the three-dimensional stress-equilibrium equations combined with finite element models based on the first-order shear-deformation theory.[4–6]

In this Note, through-the-thickness distributions of displacements as well as stresses of sandwich panels are evaluated by a postprocessing-type approach. A postprocessing predictor–corrector procedure has been utilized for the analysis of multilayered composite panels.[6] However, the procedure, if applied directly, yields inaccurate results for sandwich panels because of the large difference of material properties between the face sheets and the core. For the present study, the procedure is modified and applied to the analysis of sandwich panels with multilayered composite face sheets. The sandwich panel is subjected to both mechanical and thermal loading.

The effectiveness of the present procedure is demonstrated by means of numerical examples of the sandwich panels composed of multilayered composite face sheets and core with various elastic moduli. The solutions obtained by the present procedures are compared with the exact solutions of the three-dimensional thermoelasticity equations of the panel.[7]

II. Finite Element Model

The finite element used in conjunction with the proposed procedure is an assumed strain solid element based on the Hellinger–Reissner variational principle.[8] The present finite element model uses two unknowns: assumed displacements and independently assumed strains. The assumed strains play the role of reducing the locking effects and suppressing the spurious kinematic modes. Each element has two nodes in the thickness direction, with nine nodes on each of the top and the bottom surfaces. Each node has only three translational degrees of freedom. Rotational degrees of freedom are represented by the differences of the translational degrees of freedom at the top and the bottom surfaces. A detailed description of the formulation can be found in Ref. 8.

III. Predictor–Corrector Procedures

In the predictor procedure, through-the-thickness distributions of the in-plane stresses are evaluated by the superconvergent recovery technique presented by Zienkiewicz and Zhu.[9] Based on the in-plane stresses and utilizing the stress-equilibrium equations, the evaluation of the transverse stresses is by piecewise integration in the thickness direction of the panel. The equations for the evaluation of transverse stresses can be represented as follows:

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$$\sigma_{\gamma 3} = -\int_{-h/2}^{x_3} (\sigma_{\gamma 1,1} + \sigma_{\gamma 2,2})\,\mathrm{d}x_3 + c_\gamma \qquad (\gamma = 1, 2, 3) \quad (1)$$

where the constant c_γ is determined by the boundary conditions on the lower surface of the panel.

In the corrector procedure, through-the-thickness distributions of displacements are computed from the predicted stresses by use of the three-dimensional thermoelasticity equations:

$$\epsilon_{ij}^{\mathrm{total}} = S_{ijkl}\sigma_{kl} + \alpha_{ij}\Delta T \quad (2)$$

$$u_3^* = \int_{-h/2}^{x_3} \epsilon_{33}^{\mathrm{total}}\,\mathrm{d}x_3 \quad (3)$$

$$u_\gamma^* = \int_{-h/2}^{x_3} \left(2\epsilon_{\gamma 3} - u_{3,\gamma}^c\right)\mathrm{d}x_3 \qquad (\gamma = 1, 2) \quad (4)$$

where $\epsilon_{ij}^{\mathrm{total}}$ indicates the total strain tensor, S_{ijkl} is the flexibility tensor, α_{ij} is the tensor of the thermal-expansion coefficients, and ΔT is the temperature change. The variables with the superscript $*$ indicate quantities obtained in the corrector procedure by the strain-displacement relations and $u_{3,\gamma}^c$ is the derivative of corrected displacement u_3^c, which will be expressed in Eqs. (9).

The displacements are corrected by quantitative matching of averages and/or slopes with those of the displacements obtained from the finite element analysis. The transverse displacement (u_3) is corrected by equilibration of the average (in the face sheet part of the thickness) with that of the finite element analysis:

$$\int_{x_3^{fs}} \left(u_3^* + d_3\right)\mathrm{d}x_3 = \int_{x_3^{fs}} u_3^f\,\mathrm{d}x_3 \quad (5)$$

where the variables with the superscript f indicate quantities of the finite element analysis, the superscript fs indicates the face sheet,

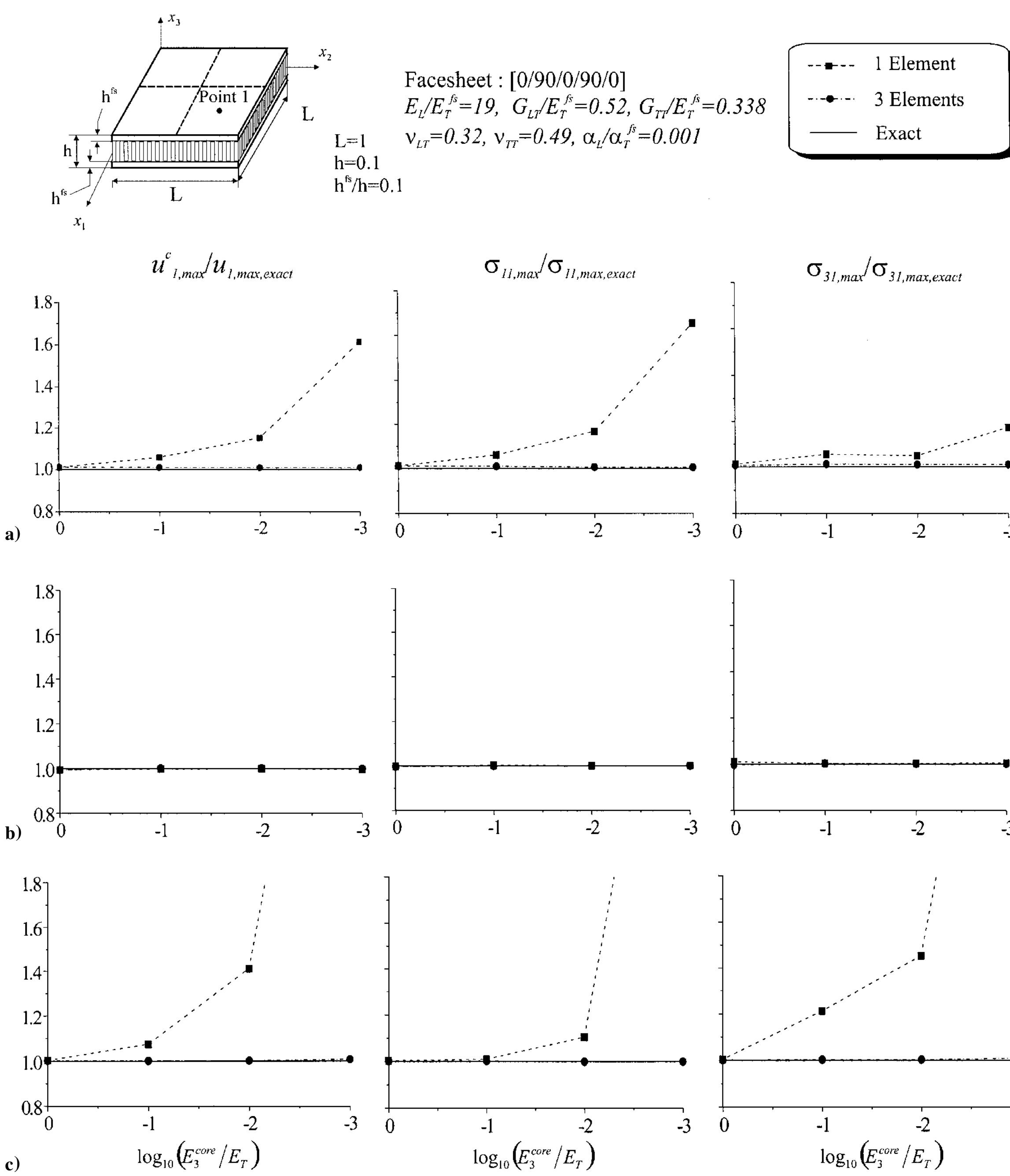

Fig. 1 Accuracy of in-plane displacement u_1, in-plane stress σ_{11}, and transverse stress σ_{31} with respect to the transverse core modulus in the sandwich panel subjected to a) transverse pressure loading, b) uniform temperature through the thickness, and c) uniform temperature gradient through the thickness.

and d_3 is the magnitude-correction coefficient. For the correction of in-plane displacements, each of the displacement components is divided into a symmetric part and an antisymmetric part with respect to the midplane of the sandwich panel:

$$u_\gamma^* = \Phi_\gamma + \Psi_\gamma \qquad (\gamma = 1 \text{ or } 2) \tag{6}$$

where Φ_γ and Ψ_γ represent the symmetric and the antisymmetric distribution of the displacements with respect to the midplane, respectively. The symmetric parts are corrected by equilibration of the average (in the face sheet part of the thickness) with that of the finite element displacements. The slopes of the antisymmetric parts are corrected by matching of the slope with that of the finite element displacements:

$$\int_{x_3^{fs}} (\Phi_\gamma + d_\gamma)\,\mathrm{d}x_3 = \int_{x_3^{fs}} u_\gamma^f\,\mathrm{d}x_3 \qquad (\gamma = 1 \text{ or } 2) \tag{7}$$

$$\frac{\mathrm{d}}{\mathrm{d}x_3}(\bar{\Psi}_\gamma + s_\gamma x_3) = \frac{\mathrm{d}u_\gamma^f}{\mathrm{d}x_3} \qquad (\gamma = 1 \text{ or } 2) \tag{8}$$

where d_γ and s_γ represent the magnitude correction coefficients and slope correction coefficients, respectively, and $\bar{\Psi}_\gamma$ represent the antisymmetric distributions of in-plane displacements of the face sheet, linearized in the least-squares sense. Then the corrected displacements can be represented as follows:

$$u_\gamma^c = u_\gamma^* + s_\gamma x_3 + d_\gamma \qquad (\gamma = 1 \text{ or } 2)$$
$$u_3^c = u_3^* + d_3 \tag{9}$$

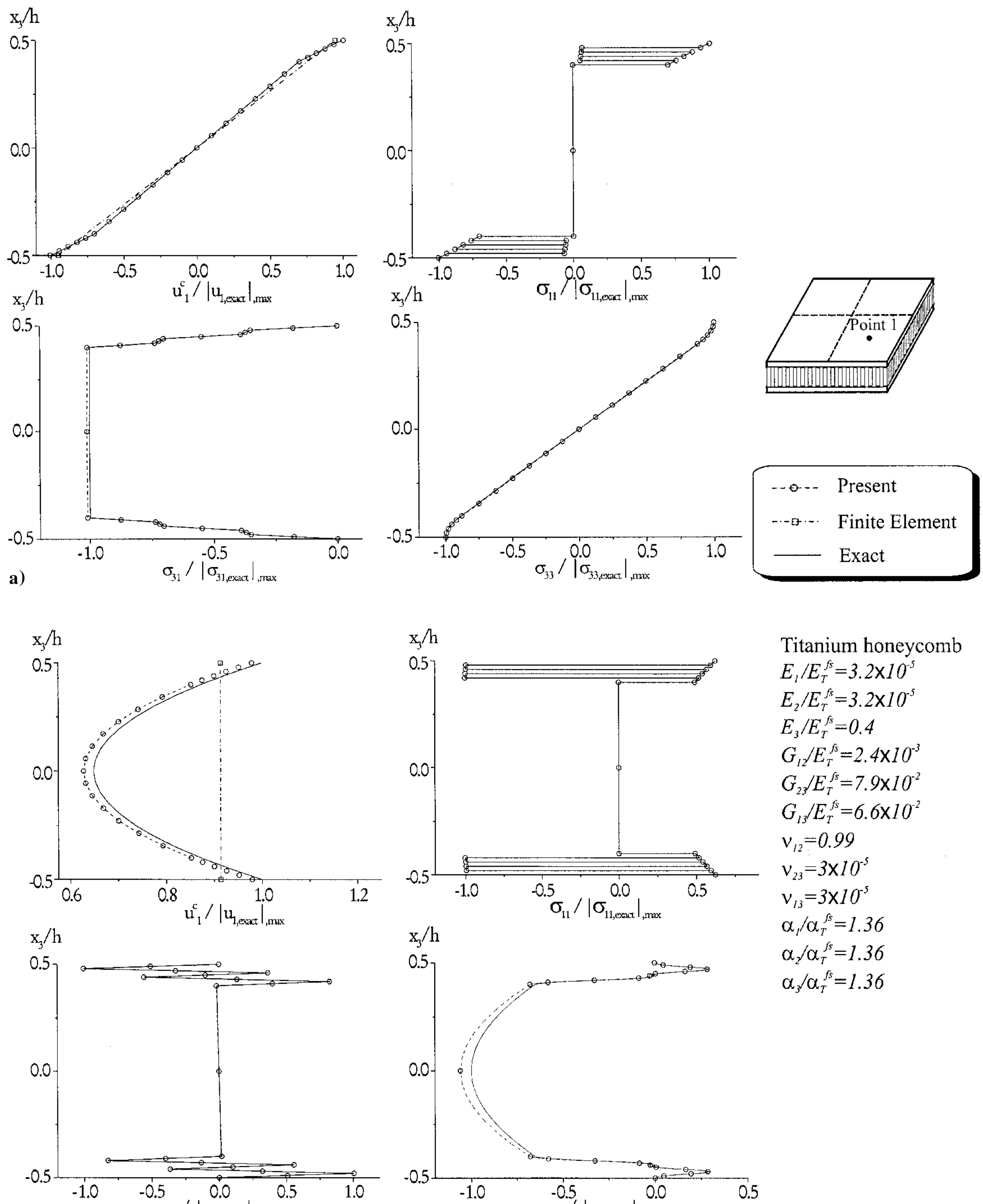

Fig. 2 Through-the-thickness distribution of in-plane displacement u_1, in-plane stress σ_{11}, transverse shear stress σ_{31}, and transverse normal stress σ_{33}; a sandwich panel with a titanium honeycomb core subjected to a) transverse pressure loading and b) uniform temperature change through the thickness.

Based on the corrected displacements, through-the-thickness distributions of strains and stresses are evaluated with the three-dimensional thermoelasticity equations and the stress-equilibrium equations:

$$\epsilon_{ij}^{\mathrm{mech}} = \tfrac{1}{2}\left(u_{i,j}^{c} + u_{j,i}^{c}\right) - \alpha_{ij}\Delta T \tag{10}$$

$$\sigma_{ij} = \boldsymbol{C}_{ijkl}\epsilon_{kl}^{\mathrm{mech}} \qquad (i, j = 1 \text{ or } 2) \tag{11}$$

$$\sigma_{\gamma 3} = -\int_{-h/2}^{x_3} [\sigma_{\gamma 1,1} + \sigma_{\gamma 2,2}]\,\mathrm{d}x_3 + c_\gamma \qquad (\gamma = 1, 2, 3) \tag{12}$$

where $\boldsymbol{C}_{ijkl}$ is the stiffness tensor, $\epsilon_{ij}^{\mathrm{mech}}$ indicates the mechanical part of the strain tensor, and the constant c_γ is determined by the boundary conditions of the lower surface of the panel.

IV. Numerical Studies

Because of large differences of moduli between face sheets and the core, analyses of sandwich panel have caused considerable difficulties.[10] In particular, when the core stiffness is considerably smaller than that of the face sheets, the deformation is quite apart from linear distribution through the thickness, which renders a finite element analysis based on classical laminate plate theory or first-order shear-deformation theory, with a single element in the thickness direction inadequate. The transverse modulus of the core has a dominant effect on the overall behavior of the sandwich panel. Thus numerical studies are performed to assess the accuracy of displacements and stresses with respect to the transverse modulus of the core. All of the computations are made at point 1, shown in Figs. 1–3. Figure 1 shows the accuracies of in-plane displacement u_1^c, in-plane stress σ_{11}, and transverse shear stress σ_{31}. The sandwich panel is subjected to one of the following loading

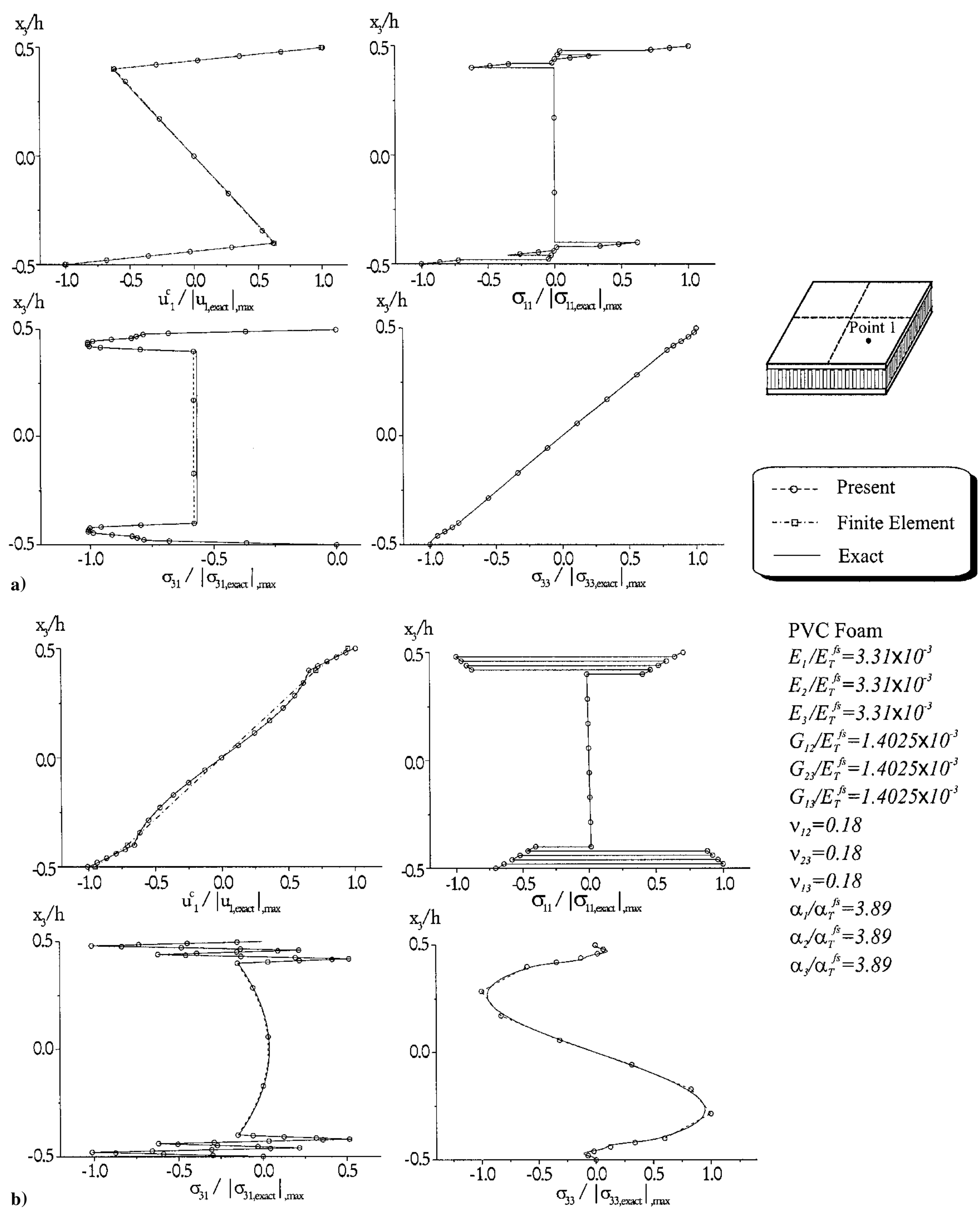

Fig. 3 Through-the-thickness distribution of in-plane displacement u_1, in-plane stress σ_{11}, transverse shear stress σ_{31}, and transverse normal stress σ_{33}; a sandwich panel with a PVC foam core subjected to a) transverse pressure loading and b) uniform temperature gradient through the thickness.

conditions: 1) transverse pressure loading with half of the loading applied at each of the top and the bottom surfaces, $p = p_0 \sin \pi \xi_1 \sin \pi \xi_2$; 2) thermal loading that is due to uniform temperature distribution through the thickness, $T = T_0 \sin \pi \xi_1 \sin \pi \xi_2$; and 3) thermal loading that is due to uniform temperature gradient through the thickness, $T = 2\xi_3 T_1 \sin \pi \xi_1 \sin \pi \xi_2$, where p_0, T_0, and T_1 are constants. Henceforth, the three cases mentioned are referred to as the $\boldsymbol{P}$, $\boldsymbol{T}_0$, and $\boldsymbol{T}_1$, cases, respectively. The results with a single element in the thickness direction are compared with those with three elements (two finite elements for the face sheets and a single element for the core) in the thickness direction. The results show that the accuracies with a single finite element in the thickness direction severely degrade as the core modulus becomes small for the $\boldsymbol{P}$ and $\boldsymbol{T}_1$ cases. However, no degradation in accuracies was observed when the sandwich panel was descretized with three elements in the thickness direction.

Through-the-thickness distributions of displacements and stresses are computed for sandwich panels with a titanium honeycomb core or a polyvinyl chloride (PVC) foam core under both mechanical and thermal loads by use of the present procedure. Two core materials chosen for the numerical analysis are typical examples of a relatively hard core and soft core, respectively. For the evaluation of a sandwich panel with a hard core, a finite element analysis was conducted with a single element in the thickness direction and yielded quite accurate results, as shown in Fig. 2. However, the sandwich panel with a soft core required three elements in the thickness direction for accurate evaluation. The results with three elements are displayed in Fig. 3.

V. Conclusion

An efficient procedure for the evaluation of displacements and stresses in sandwich panels composed of multilayered composite face sheets and core with various moduli is presented. Numerical studies are carried out to assess the accuracy of through-the-thickness distributions of displacements and stresses with respect to the various core materials. Displacements and stresses are computed with a single element in the thickness direction for a sandwich panel with a titanium honeycomb core and three elements for a sandwich panel with a PVC foam core. From the numerical studies the followings conclusions could be drawn:

1) The present predictor–corrector procedure is accurate and efficient for the evaluation of through-the-thickness distributions of displacements and stresses of sandwich panels subjected to thermal as well as mechanical loading.

2) For sandwich panels with a hard core, a single finite element is sufficient for the accurate evaluation of through-the-thickness responses.

3) Three-element modeling (two for face sheets and one for core) is required for accurate evaluation of through-the-thickness responses of sandwich panels with a soft core.

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